CONVERGENCE ENERGY IN THE ELECTRON FLOW RATE DISTRIBUTION

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Electrons in gas under an electric field are always forced to circulate the loop of a flight and a collision, and the circulation flow rate distribution autonomously settles in a steady form so as to afford the continuity of the electron flow in energy space. The steady flow rate distribution $\Psi_{sn}(\varepsilon_0)$ has a unique form determined by the energy dispersion functions $L(\varepsilon_0', \varepsilon_0)$ through the circulating loop, and gives electrons in flight a steady energy distribution. Essential is the steady circulation flow of electrons with the rate distribution satisfying the continuity in energy space, but is not the energy distribution of electrons in flight which has no self-controlling path. The electron energy distribution is only a passive state function given by the electron flow rate distribution $\Psi_{sn}(\varepsilon_0)$. The steady state in nature is self-consistently formed as above. The same process has been realized in the FTI method[1, 2], and therefore it can easily give exact data on the energy distribution and transport properties of electrons in the stationary PT (SPT) condition. This is a new idea that we believe to be true not only for electrons in gas but also in all the gas systems.

One more new acquaintance is given in this paper on the energy being exerted to maintain the flow rate distribution of electrons in a steady form. In order to understand the results of calculation clearly, it is assumed that only elastic isotropic collision occurs. A hypothetical mass ratio between an electron and a gas particle of $1/100$ is adopted to make the relaxation fast. Three collision probabilities in unit distance dependent on the powers of electron energy $Nq_0\varepsilon^r$, $[r = 1/2, 0, -1/2\ ]$ [cm^{-1}] are used to see the influence of energy dependent collisions, where q_0 is determined so that electrons may have the same mean energy in respective r values under the reduced electric field E/N of 10 Td.

In the FTI method, the loop energy dispersion functions $L(\varepsilon_0', \varepsilon_0)$, that describe the energy dispersion probabilities in passing through a trajectory motion and a collision for an electron started with energy ε_0 are prepared at first, and operated iteratively to a normalized distribution of arbitrary form as

$$\Psi_{sn}(\varepsilon_0') = L(\varepsilon_0', \varepsilon_0) \otimes \Psi_{sn}(\varepsilon_0). \quad (\varepsilon_0' \to \varepsilon_0) \tag{1}$$

Here, $\otimes$ implies the overlap integral[2]. Even started from different distributions, a steady flow rate distribution $\Psi_{sn}(\varepsilon_0)$ in normalized form is uniquely obtained as shown in Fig.1. When $\Psi_{sn}(\varepsilon_0)$ is determined, the staying time distribution $F_f(\varepsilon)$ of flowing electrons is obtained by operating the energy dispersion probabilities in flight $H_f(\varepsilon, \varepsilon_0)$ once to $\Psi_{sn}(\varepsilon_0)$.

$$F_f(\varepsilon) = H_f(\varepsilon, \varepsilon_0) \otimes \Psi_{sn}(\varepsilon_0). \tag{2}$$

Accordingly, the normalized energy distribution of electrons in flight $F_n(\varepsilon)$ in SPT condition is obtained. by normalizing $F_f(\varepsilon)$ with the mean flight time $\langle\tau\rangle = \int_0^\infty F_{f0}(\varepsilon)d\varepsilon$ as

$$F_n(\varepsilon) = F_f(\varepsilon)/\langle\tau\rangle. \tag{3}$$

Since $L(\varepsilon_0', \varepsilon_0)$ describe the probabilities of energy dispersion due to a trajectory motion and a collision for an electron started with energy ε_0, the probabilities of energy gain in a circulation are given by the energy gain function

$$LG\varepsilon(\varepsilon_0) = \int_0^\infty (\varepsilon_0' - \varepsilon_0) L(\varepsilon_0', \varepsilon_0) d\varepsilon_0' \tag{4}$$

Table 1: Basic data of electrons for 3 values of r

$Nq_0\varepsilon^r$	cm^{-1}	$40\varepsilon^{-1/2}$	$27.25\varepsilon^0$	$18.3\varepsilon^{1/2}$
$\langle\varepsilon\rangle$	eV	1.5625	1.5627	1.5646
$\langle\nu\rangle$	$10^9\mathrm{s}^{-1}$	2.3723	1.9147	1.6981
$\langle\tau\rangle$	$10^{-10}\mathrm{s}$	4.2152	5.2229	5.8890
$\langle EGx(\varepsilon_0)\rangle$	$10^{-2}\mathrm{eV}$	3.1250	3.7644	4.0589
$\langle 0.02\varepsilon'\Psi_c(\varepsilon')\rangle$	$10^{-2}\mathrm{eV}$	3.1250	3.7644	4.0589
$\langle RG\varepsilon(\varepsilon_0)\rangle_{+-}$	$10^{-2}\mathrm{eV}$	0.96316	1.7537	2.4731
$\langle G\lambda\rangle$	$10^{-2}\mathrm{cm}$	2.8692	3.6698	4.1808
W	$10^6\mathrm{cms}^{-1}$	7.4138	7.2075	6.8924

and the energy gain rate in a circulation of a flowing electron $\Psi_{sn}(\varepsilon_0)$ is given as

$$RG\varepsilon(\varepsilon_0) = LG\varepsilon(\varepsilon_0)\Psi_{sn}(\varepsilon_0). \tag{5}$$

$RG\varepsilon(\varepsilon_0)$ gives the energy being exerted for maintaining the flow rate distribution in the converged form $\Psi_{sn}(\varepsilon_0)$ in a circulation. In Fig.2, $LG\varepsilon(\varepsilon_0)$ and $RG\varepsilon(\varepsilon_0)$ calculated for three r values are shown. The positive and negative areas in $RG\varepsilon(\varepsilon_0)$ are the same showing the reciprocal relation in the convergence, and have large values for large r.

$$\langle RG\varepsilon(\varepsilon_0)\rangle = \int_0^\infty (1/2)|RG\varepsilon(\varepsilon_0)|\mathrm{d}\varepsilon_0. \quad [\mathrm{ev/cycle}] \tag{6}$$

$\langle RG\varepsilon(\varepsilon_0)\rangle$ is named the convergence energy. It is noted that the balance of energy gain and loss in a circulation is not satisfied in respective narrow energy ranges due to the energy expenditure for convergence even in steady state though achieved in the integrated value in full energy range, as is seen in Fig.2. In Table 1, some basic SPT raw data are shown for three values of r with usual accuracy in the FTI method. In which, the energy gain $E\langle Gx(\varepsilon_0)\rangle$ and the energy loss $\langle 0.02\varepsilon'\Psi_c(\varepsilon')\rangle$[2] in a circulation are the same showing the energy balance in five digits. Convergence energies $\langle RG\varepsilon(\varepsilon_0)\rangle_{+-}$ take appreciable values though smaller than $E\langle Gx(\varepsilon_0)\rangle$. The larger value of $\langle RG\varepsilon(\varepsilon_0)\rangle$ in larger r gives faster relaxation though not shown here. In these results, we feel the providence in nature.

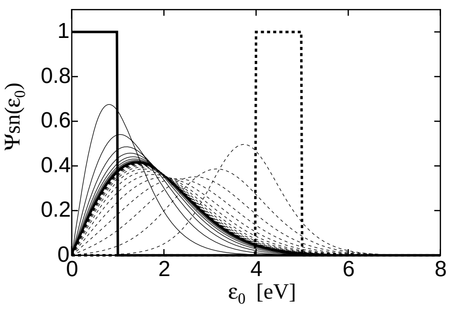

Fig. 1: Variation of $\Psi_{sn}(\varepsilon_0)$ for $r = 0$ started from two initial distributions, low mean energy (solid) and high mean energy (dotted), at every ten operations of $L(\varepsilon_0', \varepsilon_0)$.

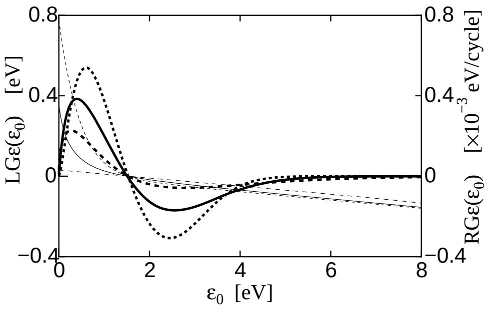

Fig. 2: Loop energy gain functions $LG\varepsilon(\varepsilon_0)$ (thin) and energy gain rates $RG\varepsilon(\varepsilon_0)$ (thick) for $r = 1/2$(dotted), 0(solid) and $-1/2$(dashed).

Reference

[1] N. Ikuta and Y. Murakami: J. Phys. Soc. Jpn. 56 (1987) 115
[2] N. Ikuta and S. Nakajima: J. Phys. Soc. Jpn. 64 (1995) 1146