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# INFLUENCE OF EXPERIMENTAL ERRORS ON PLASMA DIAGNOSTICS BY USING TIKHONOV REGULARIZATION PROCEDURES

Mira Vuceljic(*), Slavoljub Mijovic,

Faculty of Sciences, University of Montenegro, Dz. Vashingtone bb, 20000 Podgorica, Montenegro

(*) **mirav@rc.pmf.ac.me**

Many problems in plasma diagnostics [1,2] can be formulated as a linear inverse problem i.e., the problem that requires determination of unknown plasma parameters from known experimental data. Computer-supported techniques play an important role in the evaluation of experimental data but, even only discretization of inverse problems generally gives rise to very ill-conditioned linear system of algebraic equations. The ill-posed problem means that little non-avoidable errors in the measured values can lead to significant changes in the solution.

Typically, the linear systems obtained have to be regularized to make the computation of a meaningful approximate solution possible. This means that the systems must be replaced with nearby systems that is less sensitive to perturbations. Tikhonov regularization is one of the oldest and most popular regularization method. He found an effective way to regulate an ill-posed problem by using the minimum *a priori information,* as an estimation error of experimental data. The regularization method FORTRAN-subroutine has been adapted for different experimental plasma diagnostics applications whose model can be given in the operator form:

$$Az = u, \tag{1}$$

where $z-$the function we are looking for, $u-$data recorded in an experiment and $A-$a linear operator in the most cases. Tikhonov's method stabilizes least squares deviation of $Az$ from $u$ in Eq.1, using the functional $M^{\alpha}[z]$

$$M^{\alpha}[z] = \|A_{\sigma}z - u_{\delta}\|^2 + \alpha\|z\|^2, \tag{2}$$

where $\sigma$ and $\delta$ are the errors of the operator and the right part of the Eq. 1, respectively and $\alpha$ is the regularization parameter. The parameter $\alpha(\sigma,\delta)$ is determined by applying generalized discrepancy principle to an iterative method for which the functional $M^{\alpha}[z]$ attains its minimal [3].

In this work we analyzed how an estimation of the experimental errors influences the accuracy of the results obtained by the regularization method. Usually, in a real experiment the exact errors are never known, i.e., we either overestimated or underestimated it. Extensive numerical experiments are reported to demostrate the practical perfomance of the presented algorithm. As an example, we tested the problem of solving the Fredholm integral equation of the first kind

$$\int_a^b K(x,s)z(s)ds = u_0(x) + \delta(x), \qquad c \le x \le d. \tag{3}$$

where $u = u_0(x) + \delta(x)$, i.e., $\delta(x)$ is separated stationary chance process with average value equal to zero and $u_0(x)$ is the exact distribution.

Eq. 3 decribes many plasma diagnostics methods such as, obtaining real profiles of spectral lines in dense plasmas [1], electron energy distribution function from the probe measurements [2] etc. It is known that the problem (3) is ill-posed.

The procedures of the test and the regularization algorithm were following: the $z(s)$ and $K(x,s)$ in (3) were chosen and $u_0(x)$ were calculated according to Eq. 3. i.e., the forward problem was solved. Then, artificial random noises $\delta(x)$ were added to $u_0(x)$, to simulate a "measured" data with experimental errors. Finally, Tikhonov's regularization method was used to solve the inverse problem, i.e., from such "measured" data, the $z(s)$ was determined and compared with the given (exact) one. Some results are shown in Fig. 1.

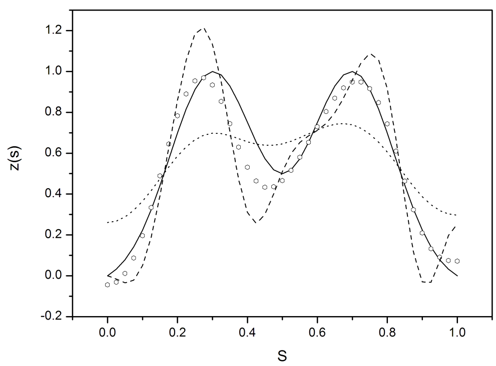


Fig. 1: Reconstructed *z(s)* for added 20% *u₀(x)* errors; solid line-exact *z(s)*, ovals-the estimated errors equals the exact errors, dot line-overestimated errors 50% *u₀(x)* and dash line-underestimated errors 7% *u₀(x)*.

As one can see from the Fig. 1, it is extremely important to estimate the experimental errors as much accurate as possible. Finally, one can conclude that this algorithm could be applied even for not very well accurate experimental data.

**Reference**

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[3] A.N.Tikhonov and A.V.Goncharski,1987 *Ill-posed Problems*, (Moscow University Press, Moscow)