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A new versatile approximation method for the line radiation description

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In plasma modelling reabsorption of radiation in spectral lines is generally neglected when it is small. However, even small values of the plasma thickness lead to a sufficient decrease of the effective escape probability $g = A_{\text{eff}}/A$, where A_{eff} and A are the effective and sponanteous transition probability, respectively. The accurate description of the plasma radiation in spectral lines requires the solution of the Holstein-Biberman equation [1–3]

$$\frac{\partial}{\partial t}N_k(\mathbf{r},t) = W_k(\mathbf{r},t) - A\left(N_k(\mathbf{r},t) + \int\limits_{(V)} N_k(\mathbf{r}',t)\frac{\mathrm{d}\mathbf{r}'}{4\pi|\mathbf{r}'-\mathbf{r}|^2}\left.\frac{\mathrm{d}T}{\mathrm{d}\rho}\right|_{\rho=|\mathbf{r}'-\mathbf{r}|}\right). \tag{1}$$

Here, N_k is the density of excited atoms and W_k represents the effective excitation due to collision processes. The kernel of the integral operator of equation (1) is expressed in terms of Biberman's transmission factor [3,4]

$$T(\rho) = \int\limits_0^\infty \varepsilon_\nu e^{-k_\nu \rho} d\nu\,, \tag{2}$$

where ε_ν and k_ν are the emission and absorption line profiles. The best way to treat radiation is the direct integration of equation (1). But it is not used in current modelling because it is not effective due to the large time required for the computation. Thus, different approximations are commonly applied. The first approach consists in the use of an asymptotic expression for the effective escape probability which is applicable in the case of large absorption coefficients. In the case of large absorption coefficient photons emitted in the centre of the line are totally absorbed and the radiation escapes only in the wings of the profile. Assuming the Lorentz form $P(\nu) = \Delta\nu/(\pi[(\nu-\nu_0)^2 + (\Delta\nu)^2])$ of the spectral line profile with the half-width $\triangle\nu$, the asymptotic for the transmission factor (2) is $T_l(\rho) = 1/\sqrt{\pi k_0 \rho}$. Here, k_0 is the absorption coefficient in the centre. $T_l(\rho)$ allows to develop exact methods for the radiation description in the case of large absorption such as resonance radiation [5–7], but it cannot be used in the case of small absorption.

In the present contribution a new approach for the description of line radiation is proposed which is based on an approximation of Biberman's transmission factor $T(\rho)$. It mainly consists of an interpolation between the asymptotic expressions for small and large absorption, where the latter is given by $T_l(\rho)$. In the case of small absorption and when the emission is weak as well, all photons are emitted primarily in the centre of the line and the part of the radiation emitted by the wings is negligible. Then, it is possible to use a simple rectangular approximation with different half-widths for the emission and absorption profiles. An appropriate approximation for the transmission factor $T_s(\rho)$ at small absorption is $T_s(\rho) = (1-\chi) + \chi e^{-k_0'\rho}$, where χ is given by $\mathrm{d}T_s/\mathrm{d}\rho|_{\rho=0} = -\chi k_0'$. Assuming a Lorentz line profile with the emission coefficient $\varepsilon_0 = 1/(\pi\Delta\nu)$ in the centre, the half-width of the corresponding rectangular emission profile has to be $\Delta\nu_{\text{re}} = \pi\Delta\nu/2$ to fulfil the criterion $\int_0^\infty \varepsilon_\nu \mathrm{d}\nu = 1$. The value of the absorption coefficient k_0' in the centre of the rectangular profile with the half-width ν_{ra} is obtained as an average value of the absorption coefficient within the limits of the rectangular emission profile according to $k_0' = (\int_{\nu_0-\Delta\nu_{\text{re}}}^{\nu_0+\Delta\nu_{\text{re}}} k_\nu \mathrm{d}\nu)/(2\Delta\nu_{\text{re}}) = k_0[\pi + \arctan(\pi/[1-(\pi/2)^2])]/\pi$ with the Lorentz profile $k_\nu = k_0\varepsilon_\nu/\varepsilon_0$. The half-width ν_{ra} is smaller than $\Delta\nu_{\text{re}}$ and is equal to $\chi\Delta\nu_{\text{re}}$, where $\chi = 0.72$.

Results of the calculated transmission factor $T(\rho)$ are presented in Fig. 1. It is found that the interpolation $T_{\text{int}} = e^{-(k_0\rho/6)^2} T_{\text{s}} + [1 - e^{-(k_0\rho/6)^2}] T_{\text{l}}$ between the asymptotic expressions for small and large absorption can be used to find a good analytical approximation for the transmission factor, which allows to develop an effective matrix method for the solution of equation (1) similar to those described in [5–7].

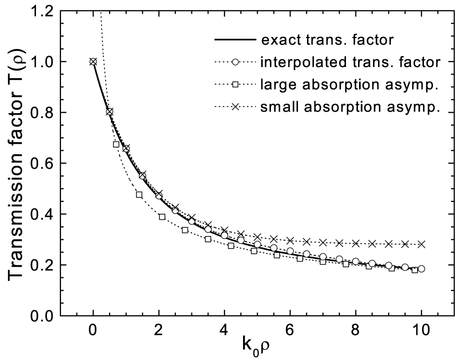

Fig. 1: Biberman's transmission factor. Solid line: exact value (2); circles: T_{int}; squares: T_{l}; crosses: T_{s}.

The results for the particle density obtained using T_{int} and T_{l}, which is accurate only in the case of large absorption, are shown in Fig. 2 for a δ-like function of the excitation source W_k in the centre of a cylinder. The comparison illustrates that the new approach should be used when the optical thickness k_0R of the plasma is less than about 5.

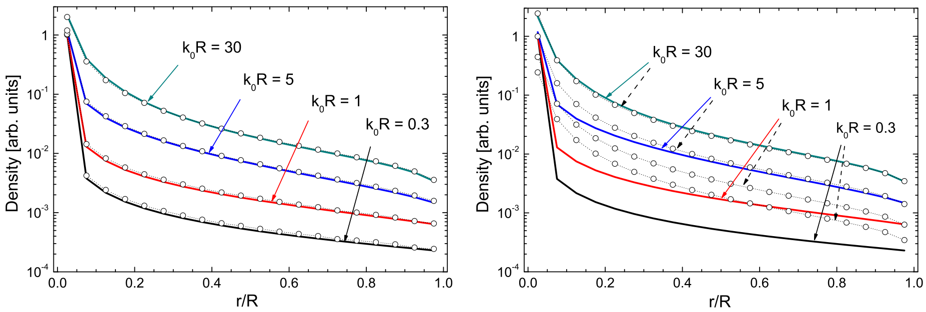

Fig. 2: Radial variation of particle density N_k. Solid lines: exact integration of equation (1); circles in left figure: T_{int}; circles in right figure: T_{l}.

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